Outcome: *Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically*

Math 8
Unit 1 – Squares and Square Roots
Square Numbers

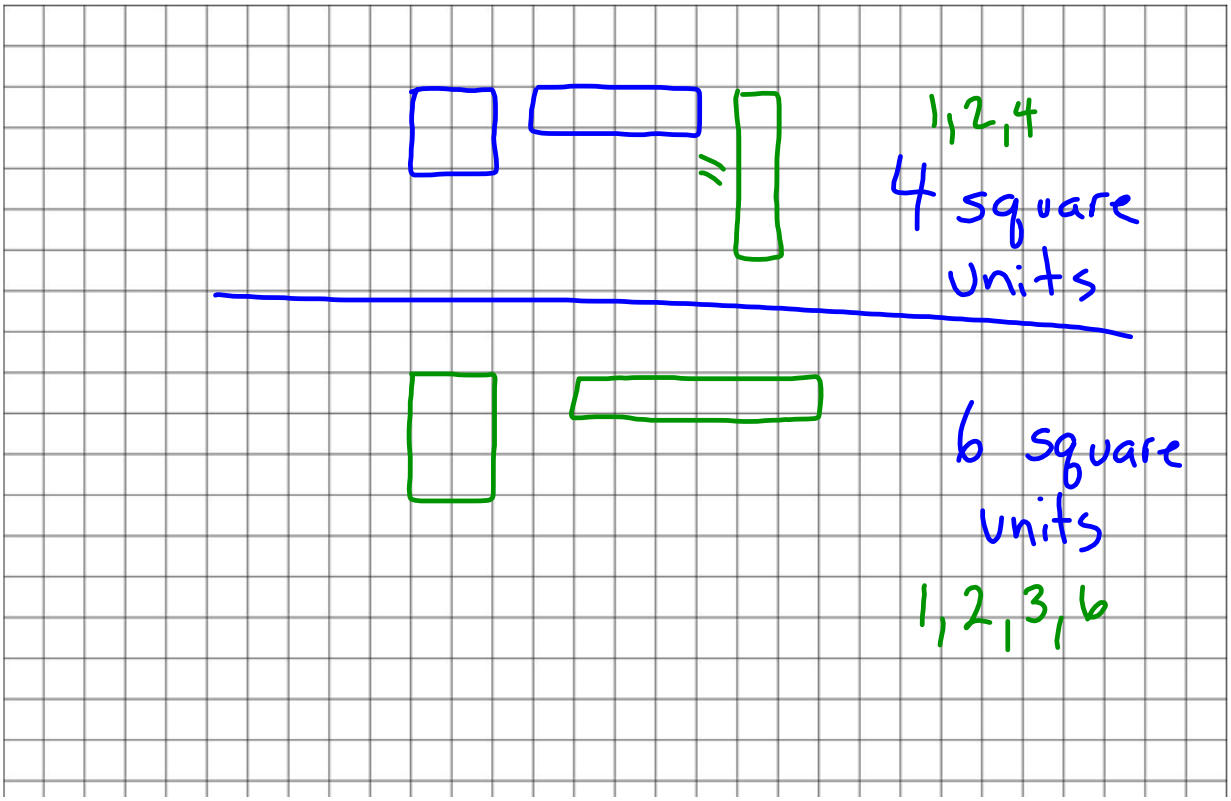
Investigate:

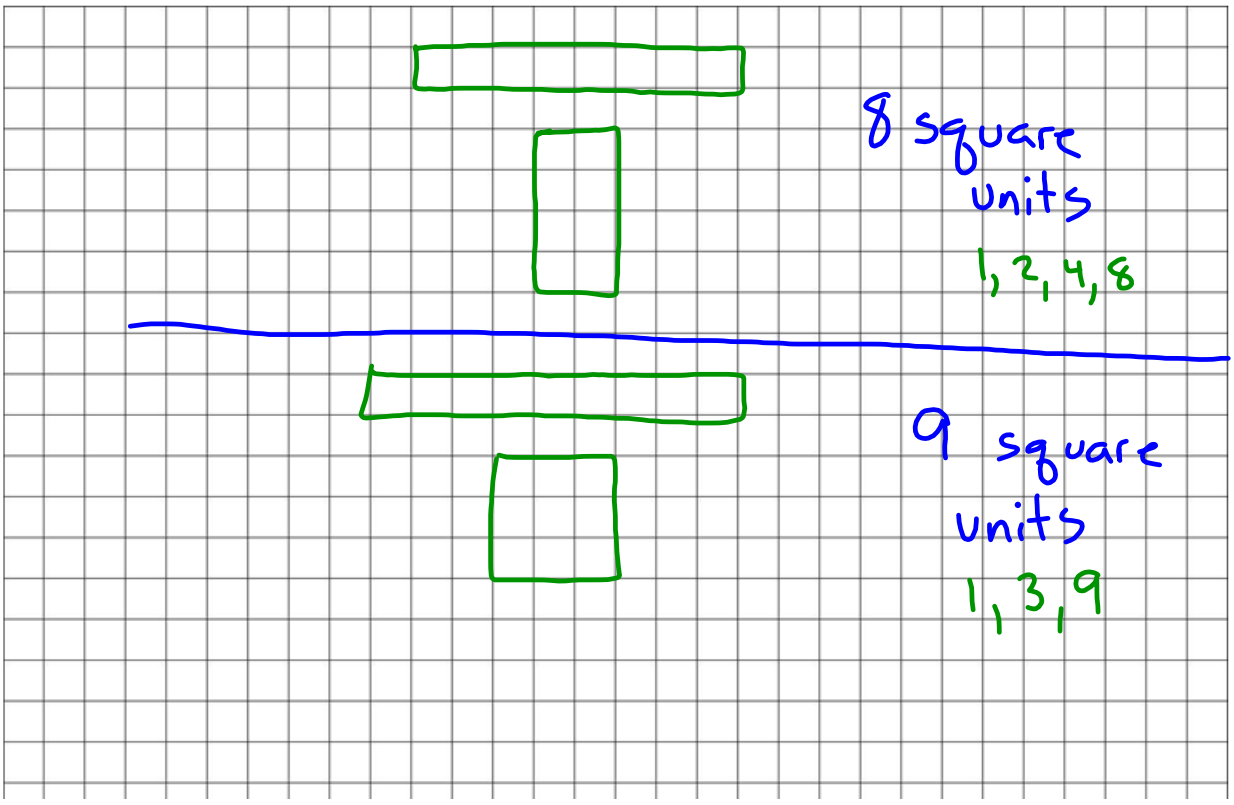
Working with a partner, using grid paper and 20 square tiles provided, make as many different rectangles as you can with each area.

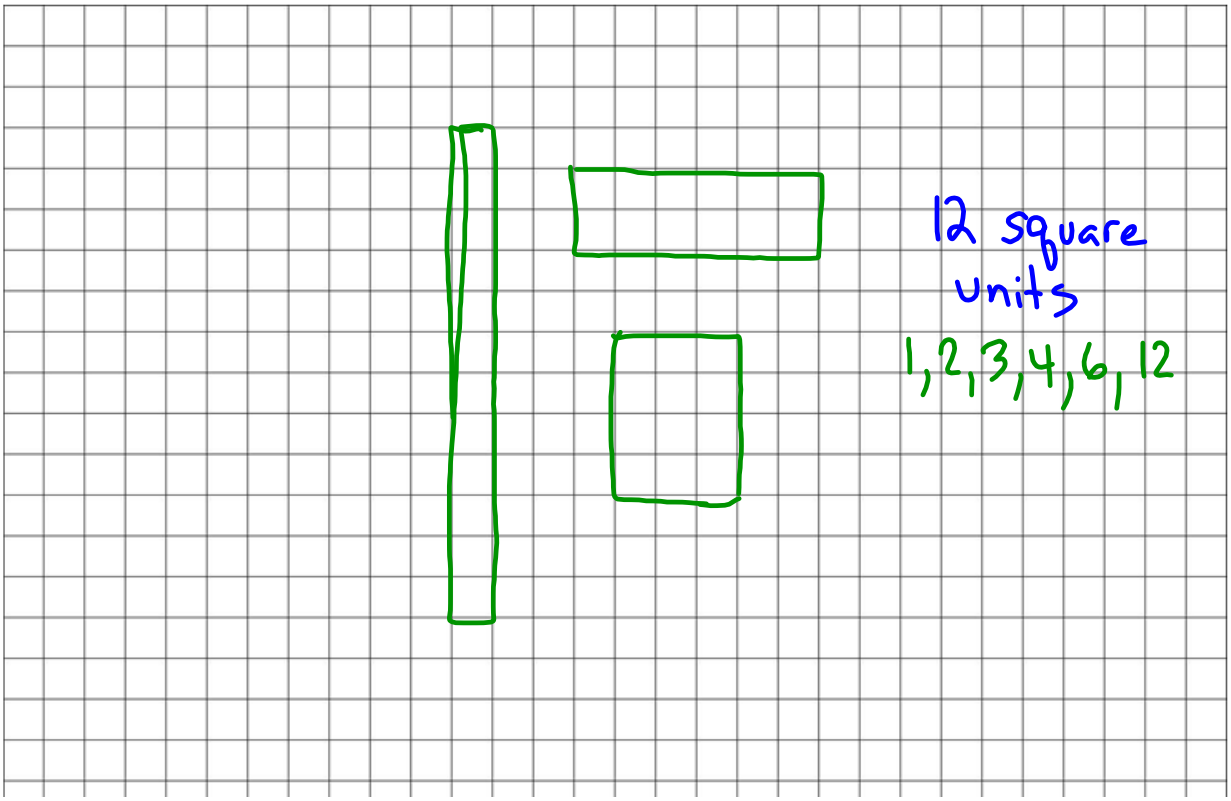
4 square units
6 square units
8 square units
9 square units
12 square units
16 square units
20 square units

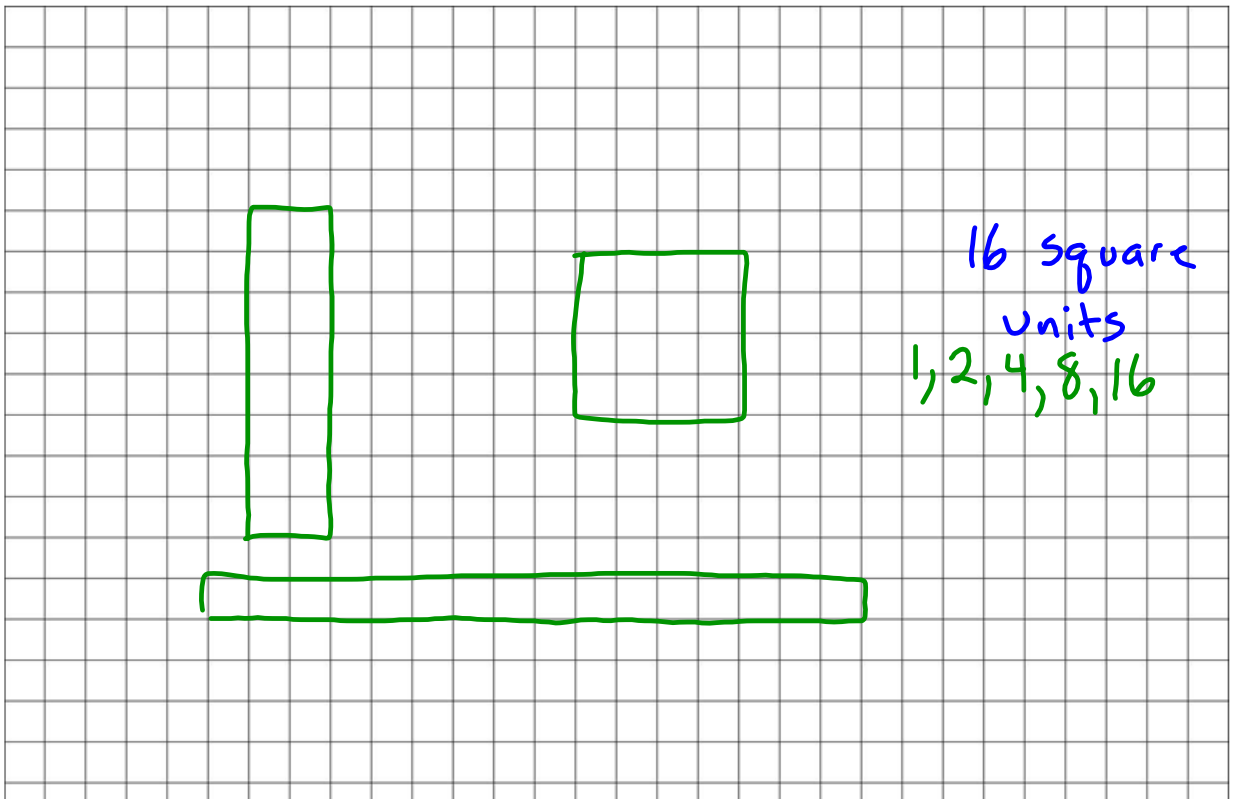
Draw the rectangles on grid paper.

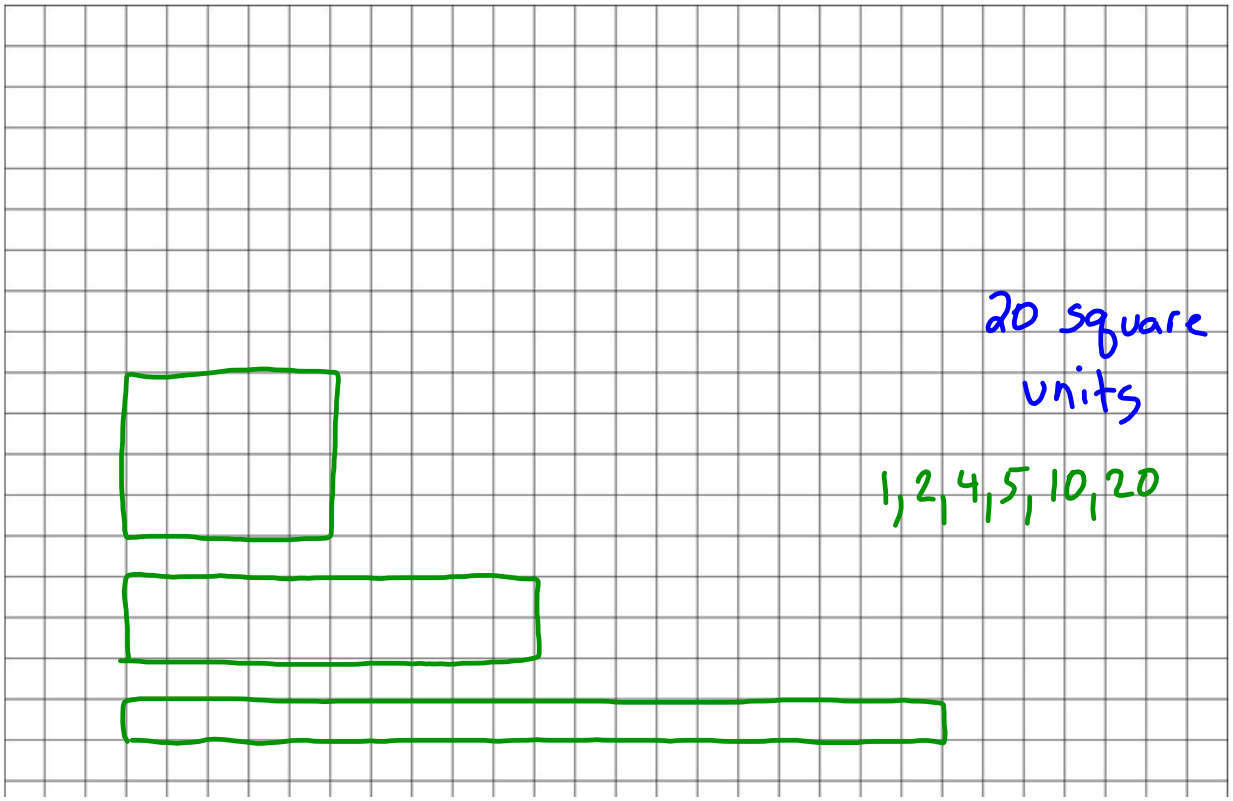
- For how many areas were you able to make a square?
- What is the side length of each square you made?
- How is the side length of a square related to its area?




8 square units
1, 2, 4, 8
9 square units
1, 3, 9


12 square units
1, 2, 3, 4, 6, 12

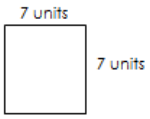

16 square units
1, 2, 4, 8, 16


20 square units
1, 2, 4, 5, 10, 20

Important Terms:

Perimeter: *The distance **around** a shape. It is measured in units.*

Ex.: A square with a side length of 7units has a perimeter of 28 units.

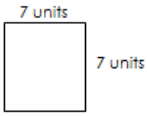


$P = l + l + l + l$ *or* $4 \times l$

P = 7 units + 7 units + 7 units + 7 units = 28 units

Area: *The amount of surface **area covered** by a shape. It is measured in square units.*

Ex.: A square with a side length of 7units has an area of 49 square units (units²).



$A = l \times w$

$A = 7units \times 7\ units = 49\ units^2$

Square Number: *The product of a number multiplied by itself. If it is a whole number multiplied by itself, it is called a* **Perfect Square (or a square number).**

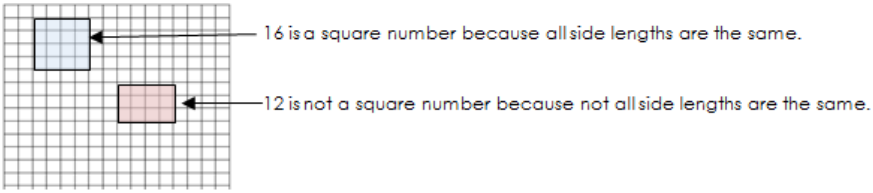
Ex.: 16 is a square number found by multiplying 4x4

Square: *The term square means to multiply a number by itself.*

Ex.: the square of 5 is 25

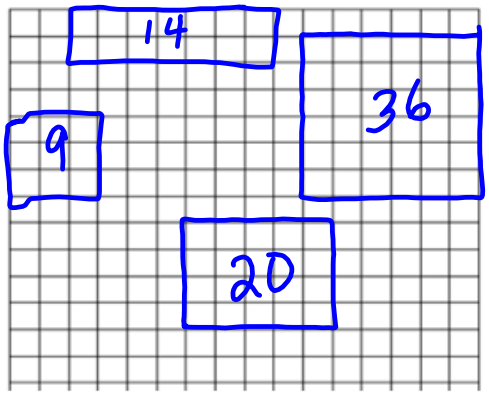
Area Model: *A model used to determine if a number is square.*

Ex.:

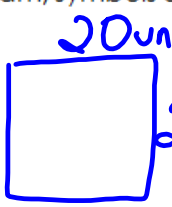


EXAMPLES:

1. Use an area model to determine if the following are square numbers:
   a. 9 ✓
   b. 14 ✗
   c. 20 ✗
   d. 36 ✓



2. Use a diagram, symbols and words to show that 400 is a square number.



$20 \times 20 = 400 \text{units}^2$

Twenty squared is four hundred.

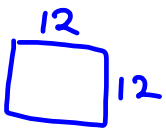
3. Find the square of the following:
   a. 2 $2 \times 2 = 4$
   b. 5 25
   c. 8 64
   d. 12 144

Your Assignment – Practice Problems:

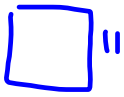
1. Use a diagram to show that each number below is a square number.

a. 1

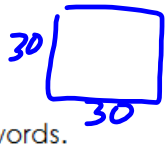
b. 144



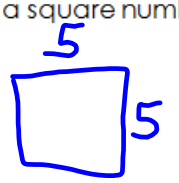
c. 121



d. 900



2. Show that 25 is a square number. Use a diagram, symbols, and words.



$5 \times 5 = 25$

Five squared is twenty-five.

3. Which of these numbers is a perfect square? How do you know?

a. 10	b. 50	c. 81	d. 20

9×9

4. Use 1-cm grid paper. Draw as many different rectangles as you can with an area of 64 cm^2. Find the base and height of each rectangle. Record the results in the following table.

Base (cm)	Height (cm)	Perimeter (cm)
1	64	130
2	32	68
4	16	40
8	8	32

a. Which rectangle has the least perimeter?

8×8

b. What can you say about this rectangle?

Square

5. I am a square number. The sum of my digits is 9. What square number might I be?

36 (3+6=9)

81

144

9

441

324

900

2025

2304

2601

3600

8100

576

360000

Square Numbers

Factoring numbers can be used to determine a perfect square. A **factor** is a number that divides exactly into another number.

Ex. 1, 2, 3, and 6 are the factors of 6.

What are the factors of 16? 1, 2, 4, 8, 16

An **odd** number of factors indicates that the number is a perfect square.

This is because The middle number is multiplied by itself.

For example:

Since 6 has 4 factors (an even number of factors) it is not a perfect square.

1, 2, 3, 6

Since 16 has 5 factors (an odd number of factors) it is a perfect square.

1, 2, 4, 8, 16

must multiply by itself

To **square** a number means to multiply the number by itself.

Example: To square 4 means 4x4

This can be expressed in ***exponential form***:

4^2

This is the **exponent**. It tells you how many times you multiply the base.

This is the **base**. It is the number that is repeatedly **multiplied**.

4^2 means 4x4

4^3 means 4x4x4

5^8 5x5x5x5x5x5x5x5

What does 5^2 mean? 5x5 = 25

Using your calculator, locate the **exponent** or **square** button. Different calculators will work in different ways.

Record the procedure for your calculator in this box:

hit the number
hit x^2 button
hit =

Try these using a calculator:

5^3 = 125

2^5 = 32

5^2 = 25

You should know the squares of 1 to 15. (This will be important when we start estimating.)

1^2	1
2^2	4
3^2	9
4^2	16
5^2	25
6^2	36
7^2	49
8^2	64
9^2	81
10^2	100
11^2	121
12^2	144
13^2	169
14^2	196
15^2	225

Know without the use of a calculator

Square Roots

A **square root** is the opposite of squaring. It is the number value for the identical set of factors that are multiplied together to make the square product. The square root is the length of the side of a square.

Example: 16 is a perfect square. It is a product of 4 × 4. The square root of 16 is 4.

16 4

This symbol represents the square root operation: $\sqrt{\square}$, so $\sqrt{25} = 5$

Try these:

$\sqrt{16}$ $\sqrt{81}$ $\sqrt{225}$

4 9 15

Using your calculator, locate the **square root button**. Different calculators will work in different ways.

Record the procedure for your calculator in this box:

Try these using a calculator:

$\sqrt{256}$ 16

$\sqrt{729}$ 27

$\sqrt{6561}$ 81

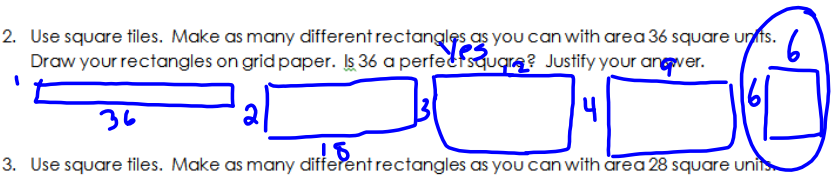
$\sqrt{1936}$ 44

$\sqrt{784}$ 28

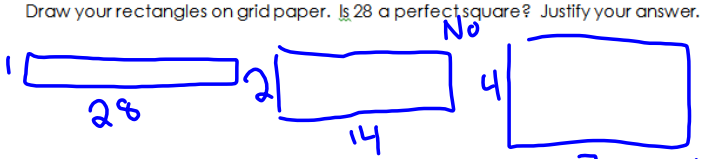
Practice with Squares and Square Roots #1

1. Find the area of a square with each side length:
   a. 8 units 64 square units 64 units^2
   b. 10 units 100 square units
   c. 3 units 9 units^2

2. Use square tiles. Make as many different rectangles as you can with area 36 square units. Draw your rectangles on grid paper. Is 36 a perfect square? Justify your answer.



3. Use square tiles. Make as many different rectangles as you can with area 28 square units. Draw your rectangles on grid paper. Is 28 a perfect square? Justify your answer.



No rectangle with side lengths 7 the same.

4. Find the square of each number.

a. 4	b. 6	c. 2	d. 9
16	36	4	81

5. Solve.

a. 8^2	b. 3^2	c. 1^2	d. 7^2
64	9	1	49

6. Find a square root of each number.

a. 25	b. 81	c. 64	d. 169
5	9	8	13

7. Complete the following questions using your calculator and patterning.
   a. Find the square of each number
      i. 1 1
      ii. 10 100
      iii. 100 10000
      iv. 1000 1000000
   b. Determine the pattern for squaring the numbers in part a. (*Hint: look at the number of zeros!) Double the number of zeros of the original number.
   c. Test your pattern by predicting the square of without a calculator, then check with a calculator.
      i. 10 000 100000000
      ii. 1 000 000 1000000000000 1×10^{12} 1 E 12
   d. Was your pattern correct?

Yes

8. List the factors of each number in ascending order. Determine if each is a square number. If it is a square number, determine the square root.

a. 256 1, 2, 4, 8, 16, 32, 64, 128, 256

b. 625 1, 5, 25, 125, 625

c. 96 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96 No

d. 441 1, 3, 7, 21, 63, 147, 441

e. 152 1, 2, 4, 8, 19, 38, 76, 152 No

f. 80 1, 2, 4, 5, 8, 10, 16, 20, 40, 80 No

9. Find each square root:

a.	$\sqrt{1}$	1	b.	$\sqrt{49}$	7
c.	$\sqrt{144}$	12	d.	$\sqrt{9}$	3
e.	$\sqrt{16}$	4	f.	$\sqrt{100}$	10
g.	$\sqrt{625}$	25	h.	$\sqrt{225}$	15

10. Find a square root of each number:

a.	~~3^1~~	3	b.	~~6^1~~	6
c.	10^2	10	d.	117^2	117

11. Find the square of each number:

a.	$\sqrt{4}$	$(\sqrt{4})^2$ 4	b.	$\sqrt{121}$	121
c.	$\sqrt{225}$	225	d.	$\sqrt{676}$	676

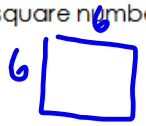
Practice with Squares and Square Roots #2

Section 1 – Complete each of the following questions **without** the use of a calculator. Show work, where applicable.

1. Complete the following table:

$1^2=$	1	$\sqrt{4}$	2
$2^2=$	4	$\sqrt{196}$	14
$3^2=$	9	$\sqrt{16}$	4
$4^2=$	16	$\sqrt{144}$	12
$5^2=$	25	$\sqrt{121}$	11
$6^2=$	36	$\sqrt{36}$	6
$7^2=$	49	$\sqrt{25}$	5
$8^2=$	64	$\sqrt{100}$	10
$9^2=$	81	$\sqrt{81}$	9
$10^2=$	100	$\sqrt{225}$	15
$11^2=$	121	$\sqrt{49}$	7
$12^2=$	144	$\sqrt{169}$	13
$13^2=$	169	$\sqrt{64}$	8
$14^2=$	196	$\sqrt{9}$	3
$15^2=$	225	$\sqrt{1}$	1

2. Show that 36 is a square number. Use a diagram, symbols and words.



$6 \times 6 = 36$ Six squared is thirty six.

3. What is the side length of a square with an area of:

a. $100m^2$	b. $225m^2$	c. $196m^2$
10m	15m	14m

$10m \mid 100m^2 \mid \sqrt{100}$

4. Which of the following numbers is a perfect square? Support your answer with an explanation.

a. 25 ✓	b. 121 ✓	c. 50 ✗ No
5×5	11×11	Sides are not same length

* Side length of a square equals the square root of the area.

5. Find the square of each number.

a. 7 — 49 b. 12 — 144 c. 15 — 225 d. 9 — 81

6. Determine the product.

a. 4^2 — 16 b. 11^2 — 121 c. 13^2 — 169 d. 10^2 — 100

7. A square number has 7 factors. Is it a square number? Explain.

Yes because it has an odd number of factors.

8. Determine the square root of each number.

a. 9 — 3 b. 16 — 4 c. 169 — 13 d. 121 — 11

9. Solve.

a. $\sqrt{225}$ — 15 b. $\sqrt{81}$ — 9 c. $\sqrt{196}$ — 14 d. $\sqrt{144}$ — 12

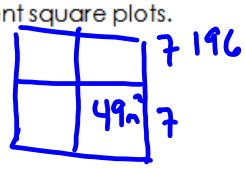
10. Solve

a. $\sqrt{5^2}$ — 5 b. $\sqrt{14^2}$ — 14 c. $\sqrt{287^2}$ — 287

11. A square garden has an area of 196m². It is divided into four congruent square plots.

a. What is the area of each plot?

$196m^2 \div 4 = 49m^2$



b. What is the side length of each smaller square plot?

$\sqrt{49m^2} = 7m$

c. What is the side length of the whole square garden area?

14m $\sqrt{196m^2} = 14m$

Section 2 – Complete the following **with** the use of a calculator. (If rounding is necessary, round to the nearest thousandth) * 3 decimal places

12. Determine the square root of:

a. 3136 56	b. 9604 98	c. 576 24

13. Solve.

a. $\sqrt{7921}$ 89	b. $\sqrt{324}$ 18
c. $\sqrt{961}$ 31	d. $\sqrt{89}$ 9.434
e. $\sqrt{165}$ 12.845	f. $\sqrt{209}$ 14.457

14. Find the square of:

a. 82 (82x82) 6724	b. 23 529	c. 109 11881

15. Solve.

a. 56^2 3136	b. 19^2 361	c. 902^2 813604

56x56

Practice with Squares and Square Roots #3

1. Find the square root of each number.
   a. 4 2
   b. 16 4
   c. 100 10
   d. 9 3

2. List the factors of each number is ascending order, then find a square root for each number.
   a. 225 1, 3, 5, 9, [15], 25, 45, 75, 225
   b. 529 1, [23], 529
   c. 144 1, 2, 3, 4, 6, 8, 9, [12], 16, 18, 24, 36, 48, 71, 144

3. Evaluate.
   a. $\sqrt{4}$ 2
   b. Find the square of 4 16
   c. 6^2 36
   d. Find the square root of 100 10

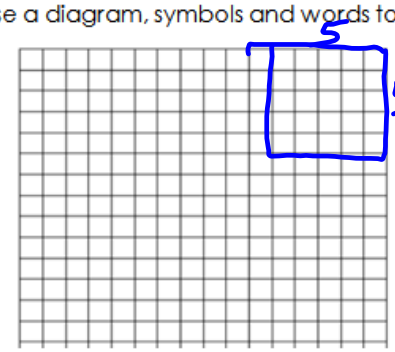
4. What is the square root of 3^2? Explain.

$\sqrt{3^2}$ square cancels square root

5. What is the square of $\sqrt{16}$. Explain.

$(\sqrt{16})^2$ 16

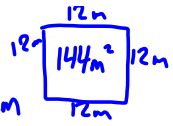
6. Use a diagram, symbols and words to illustrate the square root of 25.



$\sqrt{25} = 5$

The square root of twenty-five is 5.

7. The floor of a large square room has an area of 144m².
   a. Find the length of a side of the room.



$\sqrt{144m^2} = 12m$

   b. How much baseboard is needed to go around the room? What assumptions did you make?

No door

12m x 4 = 48m

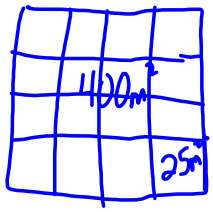
We need 48m of baseboard.

   c. Each piece of baseboard is 2.5m long. How many pieces of baseboard are needed?

48m ÷ 2.5m = 19.2 pieces You need 20 pieces.

8. A garden has an area of 400m². The garden is divided into 16 congruent square plots. Sketch a diagram of the garden. What is the side length of each plot?

$400m^2 \div 16 = 25m^2$



$\sqrt{25m^2} = 5m$

The side length of each plot is 5m.

9. Lee is planning to fence a square kennel for her dog. The area must be less than 60m², and fencing panels are sold in 1m sections.
   a. What is the kennel's greatest possible area? Explain.

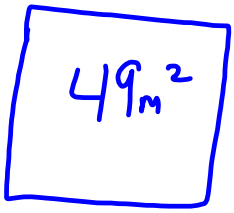
   $49m^2$

   b. Find the side length of the kennel.

   c. How much fencing is needed?

   d. One meter of fencing costs $10.00.
      i. What is the cost of the fencing?

      ii. What assumption did you make?



Outcome: *Determine the approximate square root of numbers that are not perfect squares.*

Math 8
Unit 1 – Squares and Square Roots
Practice with Benchmarks

Benchmarks are familiar values that are used as points of reference. We will use perfect squares and their square roots as benchmarks when we start estimating square roots.

Your Assignment:

For each of the following values, identify the two perfect square values that it exists between, then identify which one it is closer to. Since this is practice for estimating, ***no calculators are permitted.***

Example: 12 – *It sits between the perfect squares 9 and 16. It is closer to 9.*

1. 29 — 25 — 36 — closer to 25

2. 108 — 100 — 121

3. 37 — 36 — 49

4. 202 — 196 — 225

5. 95 — 81 — 100

6. 73 — 64 — 81

7. 90 — 81 — 100

8. 7 — 4 — 9

9. 23 — 16 — 25

10. 172 — 169 — 196

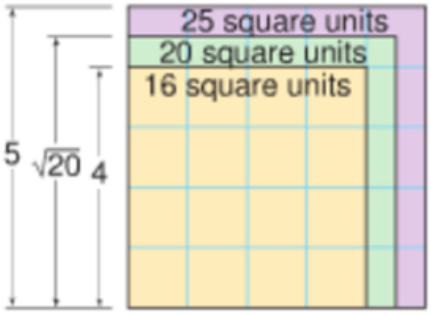
You will need to be able to estimate the square root of numbers that are **NOT** perfect square numbers.

To do this, you will need to use benchmarks and number lines.

- Remember… benchmarks are points of reference.
- For example:
    - 16 is a perfect square. It is represented by drawing a 4 x 4 square with an area of 16.
        - $\sqrt{16} = 4$ (side length of the square is 4)

    - 25 is a perfect square. It is represented by drawing a 5 x 5 square with an area of 25.
        - $\sqrt{25} = 5$ (side length of the square is 5)

    - 20 is a non-perfect square number. Because an area of 20 is between the areas of 16 and 25, you would need to have a side length between 4 and 5 to make an area of 20.
        - $\sqrt{20}$ = between 4 and 5, but you need to ***ESTIMATE***.



Estimating Non-Perfect Square Roots Using a Number Line

Example: Estimate the value of $\sqrt{20}$

1. Identify the two perfect squares that the value sits between.

benchmark **16** 20 **25** benchmark

In this case, $\sqrt{20}$ is between $\sqrt{16}$ and $\sqrt{25}$

2. Create a number line from the lower perfect square to the higher perfect square.

In this example, you would create a number line from $\sqrt{16}$ and $\sqrt{25}$.

3. Identify the value on the number line.

In this case, $\sqrt{20}$

4. Identify the square root of the two perfect squares that the value sits between. Make sure to identify these with the perfect squares on the number line. *Your estimate will sit between the square root values.*

In this case, 4 would correspond with $\sqrt{16}$ and 5 would correspond with $\sqrt{25}$. This means your value will sit between the square roots of 16 and 25, which means that your estimate for $\sqrt{20}$ will be between 4 and 5.

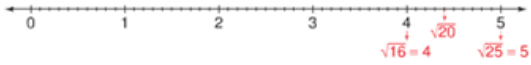
5. Identify significant points between the two perfect square values, like the halfway point between them, the quarter and three quarter points between them, etc.

6. Use these points of reference to determine the estimate to one decimal place. If it seems like it is between two values, use two decimal places.

Your Number Line for $\sqrt{20}$:

Means approximately equal to

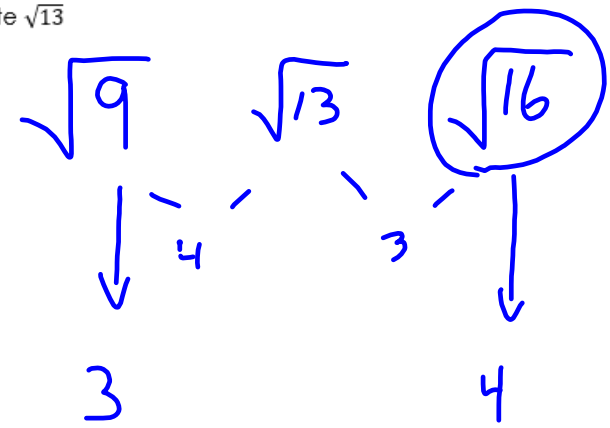
$\sqrt{20} \cong 4.45$

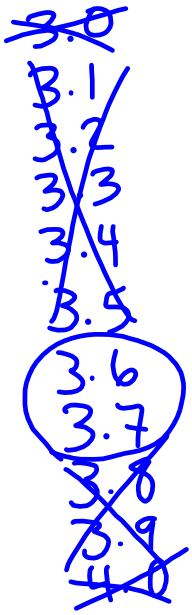


Identify the benchmarks that each of the following values sits between:

$\sqrt{1}$ $\sqrt{2}$ $\sqrt{4}$	$\sqrt{4}$ $\sqrt{5}$ $\sqrt{9}$	$\sqrt{9}$ $\sqrt{11}$ $\sqrt{16}$	$\sqrt{16}$ $\sqrt{18}$ $\sqrt{25}$
$\sqrt{16}$ $\sqrt{24}$ $\sqrt{25}$	$\sqrt{49}$ $\sqrt{56}$ $\sqrt{64}$	$\sqrt{169}$ $\sqrt{175}$ $\sqrt{196}$	$\sqrt{196}$ $\sqrt{200}$ $\sqrt{225}$

Estimate $\sqrt{13}$

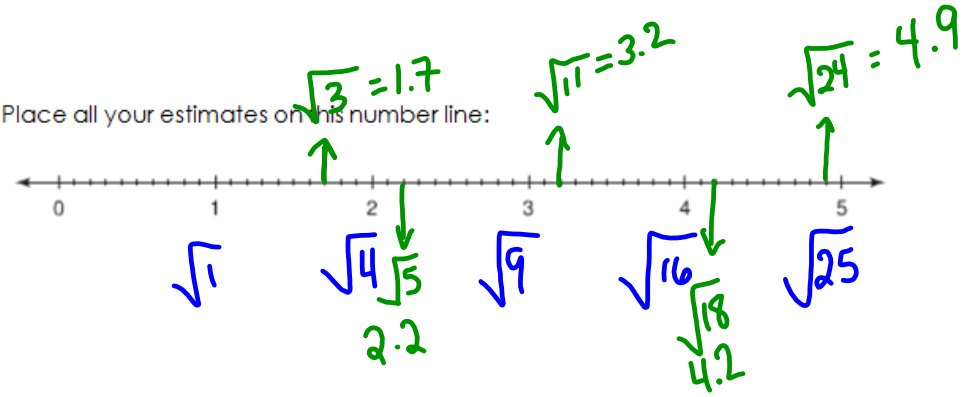




Work with a partner to estimate square roots of numbers that are not perfect. Use number lines (and grid paper if it helps). Write each estimated square root as a decimal.

$\sqrt{3}$, $\sqrt{5}$, $\sqrt{11}$, $\sqrt{18}$, $\sqrt{24}$

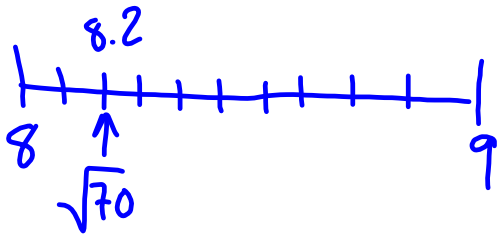
Place all your estimates on this number line:



Try these:

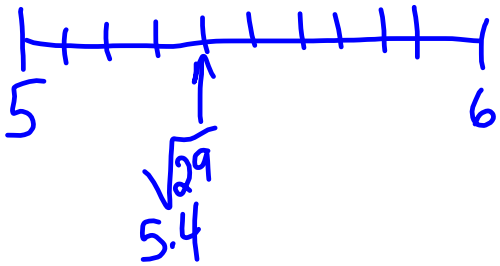
$\sqrt{64}$ $\sqrt{70}$ $\sqrt{81}$

8 9



$\sqrt{25}$ $\sqrt{29}$ $\sqrt{36}$

5 6



Practice Estimating Square Roots #1

1. Between which two consecutive whole numbers is each square root?

a. $\sqrt{4}$ $\sqrt{6}$ $\sqrt{9}$ — 2 + 3

b. $\sqrt{9}$ $\sqrt{13}$ $\sqrt{16}$ — 3 & 4

c. $\sqrt{49}$ $\sqrt{57}$ $\sqrt{64}$ — 7 + 8

d. $\sqrt{36}$ $\sqrt{38}$ $\sqrt{49}$ — 6 + 7

e. $\sqrt{171}$ — 13 + 14

f. $\sqrt{115}$ — 10 & 11

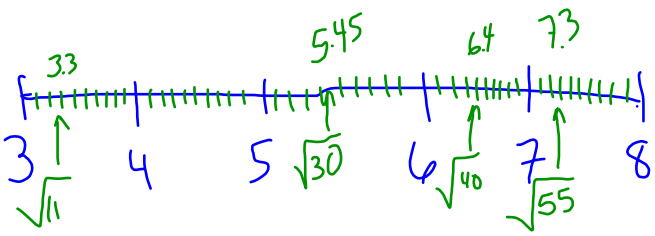
2. Draw a large sized number line from 3 to 8 (OR draw four separate number lines). Put a "tick" to show every tenth (0.1) on the line. Place each square root on the number line to show its' approximate value. You may need to complete this on a separate sheet of paper.

a. $\sqrt{11}$ $\sqrt{9}$ $\sqrt{16}$

b. $\sqrt{40}$

c. $\sqrt{30}$

d. $\sqrt{55}$



3. In each pair, is the given whole number greater than, less than, or equal to the square root? Justify your answer.

a. $7, \sqrt{14}$ greater than → $\sqrt{14}$ is between 3 + 4

b. $8, \sqrt{60}$ greater than → $\sqrt{64} = 8$

c. $11, \sqrt{121}$ equal

d. $12, \sqrt{150}$ less than → $\sqrt{144} = 12$

4. Which whole number is each square root closest to? How did you know? **Explain.**

a. $\sqrt{58}$ $\sqrt{49}$ $\sqrt{64}$ 8 - because 58 is closer to 64 than 49

b. $\sqrt{70}$ $\sqrt{64}$ $\sqrt{81}$ 8

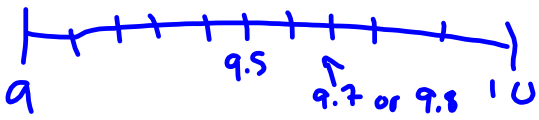
c. $\sqrt{90}$ $\sqrt{81}$ $\sqrt{100}$ 9

d. $\sqrt{151}$ $\sqrt{144}$ $\sqrt{169}$ 12

5. Find the approximate side length of the square with each area using a number line. Give each answer to one decimal place.

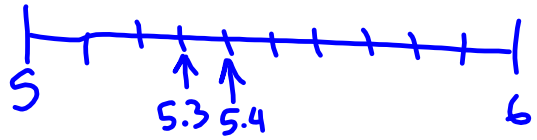
a. Estimate.

i. 92cm² $\sqrt{81}$ $\sqrt{100}$



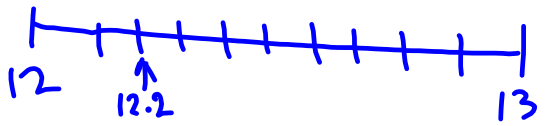
9.6

ii. 29m² $\sqrt{25}$ 5 $\sqrt{36}$ 6



5.4

iii. 150cm² $\sqrt{144}$ 12 $\sqrt{169}$ 13



12.2

b. Calculate the side length with a calculator. Record your answer to the nearest tenth.

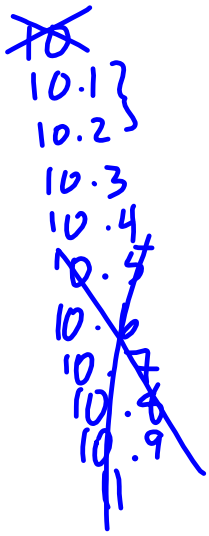
Practice Estimating Square Roots #2

Estimate the square root of each number. Make sure to use a number line and show all benchmarks. When you are done, check your answer with a calculator.

64 1. 72 81

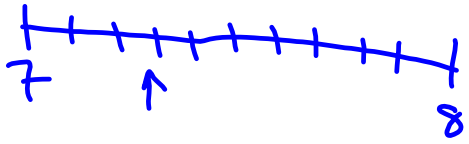
Estimate: 8.4 Actual: 8.49

100 2. 103 121



Estimate: 10.1 Actual: 10.1

49 3. 55 64



Estimate: 7.3 Actual: 7.4

9 4. $\sqrt{14}$ 16

Estimate: 3.8 Actual: 3.7

5. $\sqrt{86}$

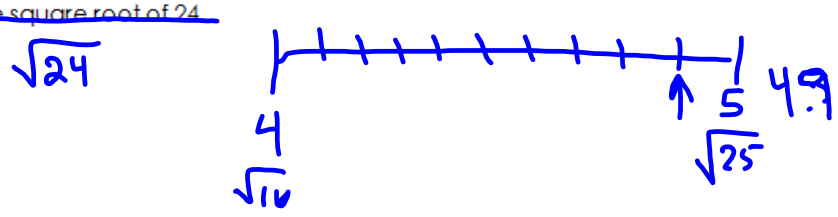
Estimate: 9.3 Actual: 9.3

6. $\sqrt{136}$ 121 144

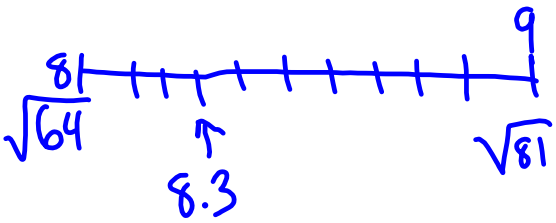
Estimate: 11.7 Actual: 11.6

Practice Estimating Square Roots #3

1. Estimate each of the following. You need to use a labeled number line with all benchmarks.
   a. Estimate ~~the square root of 24~~



   b. Estimate $\sqrt{70}$



2. An unknown number has a square root between 2 and 3. What is one possible value for the unknown number? Explain how you knew. (/2)

$\sqrt{4}$ $\sqrt{9}$ 6, 5, 7, 8

2 3 4.2

3. Determine the square root of the following using a calculator. (/4)

a. $\sqrt{62}$ 7.9

b. $\sqrt{104}$ 10.1

c. $\sqrt{19}$ 4.4

d. Explain why the answers in a – c are actually approximations and not exact answers.

We rounded the numbers.