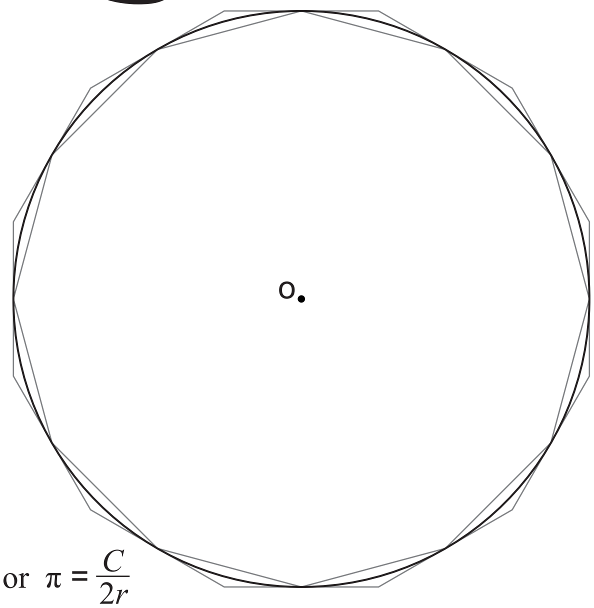
Calculating Pi



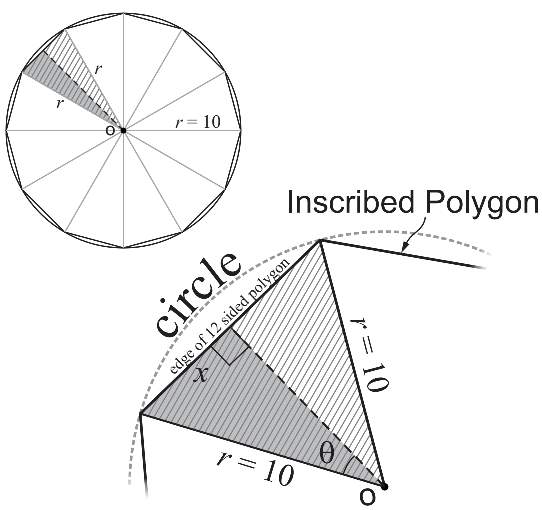
Archimedes’ Method

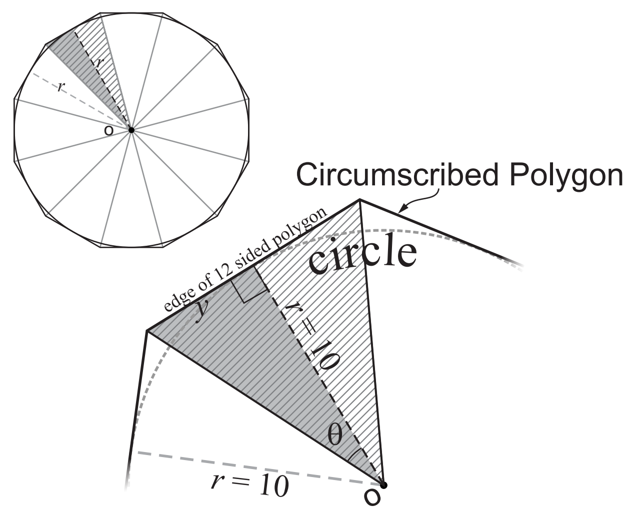
The first rigorous mathematical calculation of π was due to Archimedes (250? BC), who used a brilliant scheme based on doubling inscribed and circumscribed polygons, 6→12→24→48→96 sided polygons, and computing the perimeters.

A circle with two 12-sided polygons is shown on the right. One of the polygons is inscribed in the circle and the other is circumscribed. The objective is to find the perimeters of each of the two polygons. The average of the two perimeters is a close approximation to the circumference of the circle.

Once an approximation of the circumference of the circle is found it can be used to calculate a value for π, since $C = 2\pi r$ or $\pi = \dfrac{C}{2r}$

If more than 12 sides are used for the polygons, then the approximation for π becomes closer to the actual value of π.





1. Determine the value of angle θ for each of the diagrams.

2. Calculate the value of x and y using trigonometry.
   (let the radius be 10 units.)

3. Double the value of x to find the length of one side of the 12-sided polygon. Use this value to determine the perimeter of the inscribed 12-sided polygon.

4. Double the value of y to find the length of one side of the 12-sided polygon. Use this value to determine the perimeter of the circumscribed 12-sided polygon.

5. Calculate the average perimeter of the two polygons. This as an approximation of the circumference of the circle.

$$\text{approximate circumference of circle} = \text{average of the two perimeters} = \frac{\left(\text{perimeter of inscribed polygon} + \text{perimeter of circumscribed polygon}\right)}{2} =$$

6. Using the approximate circumference, C, of the circle from #5, calculate the value of π using the formula:

$$\pi = \frac{C}{2r}$$

$$\pi = \frac{C}{2(10)} = \frac{C}{20} =$$

7. Redo #1 – #6 using a 96-sided polygon.

Final notes:

Archimedes π approximations

6 sides	3.23205
12 sides	3.16061
24 sides	3.14614
48 sides	3.14272
96 sides	3.14187
actual	3.14159265358979323 8...

Archimedes (250 BC?) had no calculator, decimals, or trigonometry concepts. He used fractions to calculate the perimeters of the two inscribed and circumscribed polygons (96 sides). He concluded that π was between: $3\frac{10}{71} < \pi < 3\frac{10}{70}$